

# Tunable Optical Filter Using Multilayer Photonic Structure Based on Kronig-Penny Model

## Abstract

A method to design the tunable optical band pass filter using one-dimensional multilayer photonic structure has been proposed. The multilayer structure has been proposed choosing Si as high refractive index material and SiO<sub>2</sub> as low refractive index material. The refractive index of both the materials has been taken as wavelength and temperature dependent. So the allowed bands can be tuned by varying the temperature without changing geometry and angle of incidence. The proposed model is analogous to Kronig-Penny model in solid state physics.

**Keywords:** Multilayer Structure, Optical Filter, Kronig-Penny Model Etc.

## Introduction

Photonic Band Gap materials, also known as photonic crystals, are materials which have alternate forbidden and allowed band gaps. The fabrication of photonic crystals can be possible in one, two, or three dimensions. The forbidden band gap in photonic crystals represents the frequency/wavelength range where wave behaving photons cannot be transmitted through the material. An optical filter is a device which stops and/or allowed some frequency/wavelength range. It is demonstrated theoretically that these allowed/forbidden bands can be tuned as the function of temperature.

## Aim of the Study

Photonic crystals are attractive optical materials for controlling and manipulating light flow. This study is more physically realistic because thermo-optic as well as thermo-expansion effect are considered simultaneously in this study. Tuning of photonic band are useful in optical devices e.g. monochromator, optical switches etc.

## Review of Literature

Since last two decades, many researchers paid considerable much attention to photonic band-gap (PBG) materials, the artificial material which consists of periodic dielectric components in the nano- and micro-meter scale regions [1-8]. It is well known that periodic structure of materials of different refractive indices can give rise to the photonic band gaps. Such materials now-a-days are known as photonic crystals (PCs). Such materials have been investigated intensively by many investigators as to their properties like their ability of controlling the propagation of light, suppression of spontaneous emission, etc. [9-11]. Tunable optical filters have received much attention due to their application in fibre-optic communications and other fields of optical technology [12-15]. Fabrication of filters in the near- and far- infrared region was suggested by Ojha et al. [16]. This model was based on weak guidance approximation such that

$$\left( \frac{n_1 - n_2}{n_1} \right) \ll 1 \text{ and working principle was based on Kronig-Penny}$$

model in the band theory of solids. Chen et al. [17] suggested the design of optical filters using photonic band-gap air bridges and calculated important results regarding filtering properties. Recently D'Orazio et al. [18] have fabricated the photonic band-gap filter for wavelength division multiplexing. Also, Ojha et al. [19-21] gave the idea for fabrication of band pass filter in ultraviolet region, operating characteristics of metallic optical filter and optical filter in visible and infrared region. Recently, Banerjee et al. [22] designed a nano-layered tunable band pass filter by changing the incidence angle of light and changing the value of the lattice parameters.



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The tuning of all the previous filters was based on either changing the geometry or changing the angle of incidence. As we change the angle of incidence the width of a particular allowed band also increase from the same allowed band at previous angle of incidence [22]. This is the main drawback of these filters. In the present study we designed the tunable filter which works in the infrared region and allowed bands can be tuned as the function of the temperature. The band widths of a particular bandfound almost constant at each temperature.

**Theoretical analysis**

Let us consider optical transmission of electromagnetic (EM) wave through the photonic crystals having the periodicity in refractive indices of alternate materials analogues to the transmission of electron through a periodic potential in solid state electronics. For electron wave in periodic lattice, there are some allowed bands for energy states and some forbidden energy bands. If the electron waves are referring to the EM waves and the periodic atomic lattice is replaced by periodic refractive index array, allowed and forbidden bands for energies can be replaced by allowed and forbidden bands of frequencies of wavelength. By choosing a linearly periodic refractive index profile with appropriate structural parameters in the filter material one obtains a given set of frequency/wavelength ranges that are allowed or forbidden to pass throughout the proposed structure. A periodic step function can be considered in the following manner:

$$n_i(x) = \begin{cases} n_1, & 0 \leq x \leq a; \\ n_2, & -b \leq x \leq 0; \end{cases} \quad (1)$$

Where  $n_i(x) = n_i(x + md)$  with the translation factor  $m$ , where  $m = 0, \pm 1, \pm 2, \pm 3, \dots$ , and  $d = a + b$  is the lattice constant with  $a$  and  $b$  are the thickness of the two regions having refractive indices ( $n_1$ ) and ( $n_2$ ) respectively. The refractive index profile in the

$$\cos(K \cdot d) = \cos(\alpha a) \cdot \cos(\beta b) - \frac{1}{2} \left( \frac{n_1 \cdot \cos \theta_1}{n_2 \cdot \cos \theta_2} + \frac{n_2 \cdot \cos \theta_2}{n_1 \cdot \cos \theta_1} \right) \cdot \sin(\alpha a) \cdot \sin(\beta b) \quad (3)$$

For normal incidence the equation (3) can be written in the form of wave vectors as

$$\cos(K \cdot d) = \cos(\alpha a) \cdot \cos(\beta b) - \left( \frac{n_1^2 + n_2^2}{2n_1n_2} \right) \cdot \sin(\alpha a) \cdot \sin(\beta b) \quad (4)$$

where  $\alpha = \frac{2\pi n_1}{\lambda}$  and  $\beta = \frac{2\pi n_2}{\lambda}$  (5)

Now, abbreviating the L.H.S. of Equation (4) as  $L(\lambda)$ , Equation (3) may be written in the following manner:

$$L(\lambda) = \cos(K \cdot d) \quad (6)$$

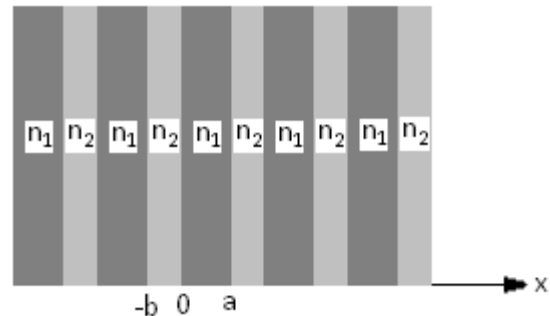
Due to the cosine function in R.H.S. of Equation (6) the maximum and minimum values of right hand side of equation (6) will be +1 and -1 respectively.

**Proposed Structure and Structural Parameters**

From the computation of Equation (4), the filtering properties of one-dimensional photonic crystals simultaneously considering thermal

graphical form of the materials is shown in the Figure 1.

**Figure 1: Periodic Refractive Index Profile of Proposed Structure**



If  $\theta_i$  is the angle of incidence for the incident EM wave on the periodic structure, the 1-D wave equation for the spatial part of the EMeigen mode

$\psi_k(x)$  is given by

$$\frac{d^2 \psi_k(x)}{dx^2} + \frac{n_i^2(x) \cdot \cos^2 \theta_i \cdot \omega_k^2}{c^2} \psi_k(x) = 0, \quad (2)$$

Where  $n_i(x)$  is given by refractive index profile as given in Equation (1),  $\theta_i$  is wave angle and  $i=1,2$  stands for the mediums of refractive index  $n_1$  and  $n_2$  respectively.

The periodic nature of the structure allows the application of Bloch's theorem where the solution of equations (2) can be written in the form of periodic function as  $\psi_K(x) = u_K(x)e^{iKx}$  where  $K$  is known as Bloch wave number and  $u_K(x)$  is the Eigen function. Thus, applying Bloch's theorem and after some mathematical exercise, the following relation can be obtained [22].

expansion effect (effect of temperature on thickness) and thermal-optic effect (effect of temperature on refractive indices) can be represented graphically. For this purpose, the PC structure having  $n_1$  and  $n_2$  as refractive indices of  $\text{SiO}_2$  and  $\text{Si}$  respectively ( $n_1 < n_2$ ) has been considered.  $\text{Si}$  and  $\text{SiO}_2$  are the good candidates for designing the photonic crystals especially in infrared region because of these materials have very low absorption in this region. The refractive indices of both the materials are taken as simultaneously considering variation in thickness and refractive index in the following manner

$$n_1(T) = n_1(1 + \alpha_1 \Delta T) \text{ and } n_2(T) = n_2(1 + \alpha_2 \Delta T) \quad (7)$$

Where  $n_1=1.5$ ,  $n_2=3.7$  (refractive indices in infrared region) and  $\alpha_1, \alpha_2$  are called the thermo-optic

coefficients for the SiO<sub>2</sub> and Si materials respectively. The values of these coefficients are chosen as  $\alpha_1=1.86 \times 10^{-4} /^\circ\text{C}$  and  $\alpha_2=6.8 \times 10^{-6} /^\circ\text{C}$  [23]. The refractive index of SiO<sub>2</sub> layers are increase more per degree of temperature in comparison of Si layers. So refractive index contrast is decreased with temperature. The property of the proposed structure can be employed to tune the ODR bands.

The thickness of each layer is taken considering the effect of temperature on thickness of both layers as follows

$$a=a(1+\beta_1\Delta T) \text{ and } b=b(1+\beta_2\Delta T) \quad (8)$$

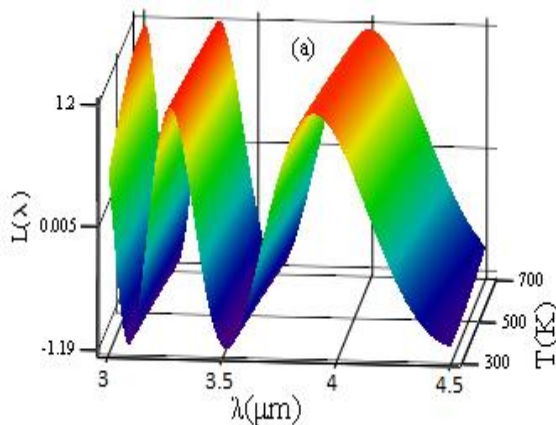
Where  $\beta_1, \beta_2$  are called the thermal expansion coefficients for the SiO<sub>2</sub> and Si layers respectively. The values of these coefficients are chosen as  $\beta_1=2.6 \times 10^{-6} /^\circ\text{C}$  and  $\beta_2=0.5 \times 10^{-6} /^\circ\text{C}$  [23].

**Results and discussions**

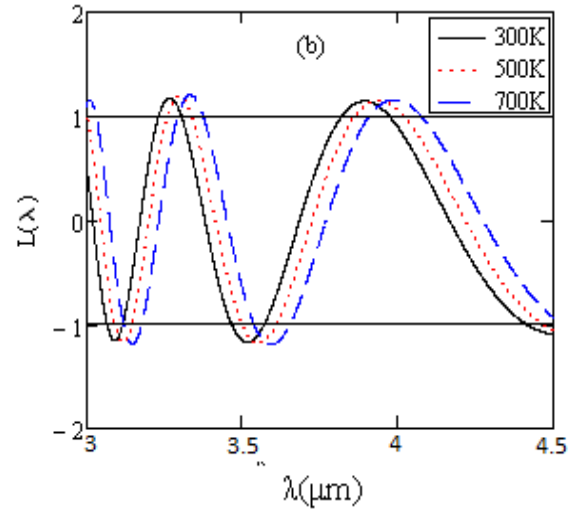
The temperature and wavelength dependent refractive indices for Si and SiO<sub>2</sub> are taken as equation (7) and (8) respectively. From equation (7) and (8), it is clear that the refractive index of these two materials is temperature and wavelength dependent. Therefore the variation in the refractive index can be used to tune the allowed bands. The thickness of both layers is taken to be equal.

It is clear from Equation (6), that the values of the expression on the right hand side of this equation are between  $\pm 1$ . Hence, the region for which  $L(\lambda)$  lying between  $\pm 1$ , is called allowed region of  $\lambda$  and for which  $L(\lambda)$  lying outside  $\pm 1$  is called forbidden region of  $\lambda$ . The wavelengths corresponding to the allowed ranges are transmitted through the filter structure but the wavelengths corresponding to the forbidden ranges are reflected from the filter structure if the absorption part is negligible. Using these parameters, the function  $L(\lambda)$  given by Equation(6) is plotted against the wavelength and temperature. The resulting curve is shown in the figures 2 and 3. The 2D plots for the temperatures 300K, 500K and 700K are shown in Figure 3.

**Figure 2: Variation of L (λ) with Temperature and Wavelength**



**Figure 3: Variation of L (λ) With Wavelength at Different Temperatures**



From the study of these Figures it is clear that for the fixed values of  $a, b, n_1$  and  $n_2$ , the allowed photonic band width increases as wavelength increases at constant temperature. As we increase the temperature, the allowed bands shift towards the higher wavelength region. But the band width of the allowed bands does not change with temperature. The whole bands shift towards higher wavelength region. It is the advantage of this study over previous one [22]. This shifting behavior of allowed bands can be explained by the equation (5). As refractive index increases with temperature in R.H.S. of equation (5),  $\lambda$  must increase to keep the L.H.S. unchanged. So we can tune the particular allowed band to a desired region by varying the temperature.

**Conclusion**

This type of filter can be used in many optical devices such as temperature sensor, wavelength demultiplexer etc and in other optical systems. By choosing appropriate values of temperature, we can design a frequency selector or rejecter. Also, by cascading two, three or more filters we can design a perfect mirror or monochromator.

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